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The Q^2 dependence of the measured asymmetry A_1 from the similarity of the $g_1(x, Q^2)$ and $F_3(x, Q^2)$ structure functions

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Abstract. We propose a new approach for taking into account the Q^2 dependence of measured asymmetry A_1. This approach is based on the similarities between the Q^2 behaviours and between the shapes of the spin-dependent structure function $g_1(x, Q^2)$ and spin-averaged structure function $F_3(x, Q^2)$. The analysis is applied to available experimental data.

1 Introduction

In recent years there has been significant progress in the study of the spin-dependent structure function (SF) $g_1(x, Q^2)$ (see [1]–[6]). The direct measurement of SF g_1 is a very elaborate procedure (see, however, [7]), and usually its value is extracted from the spin-dependent asymmetry A_1 (see, for example [8,9]) in agreement with the formula

$$g_1(x, Q^2) \;=\; A_1(x, Q^2) \cdot F_1(x, Q^2), \tag{1}$$

where $F_1(x, Q^2)$ is the spin-averaged SF.

The asymmetry $A_1(x, Q^2)$ is closely connected with the ratio of polarized and unpolarized cross-sections and may be successfully measured with the cancellation of many experimental uncertantities. Experimentally, asymmetry is extracted only at a few points $Q^2_{1i}, ..., Q^2_{ni}$ for each x_i bin. To study the properties of $g_1(x, Q^2)$ and to calculate the values of spin-dependent sum rules ([10,11]) we have to know A_1 as a function of Q^2.

The most popular assumption applied to A_1 ([12]) is

$$A_1(x, Q^2) = A_1(x). \tag{2}$$

This means that SFs g_1 and F_1 have the same Q^2 dependences, but this conclusion does not follow from the theory. On the contrary, the behaviours of F_1 and g_1 as functions of Q^2 are expected to be different due to the difference between polarized and unpolarized splitting functions[1].

There exist several approaches ([13]-[18]) to take into account the Q^2 dependence of A_1. They are based on different approximate solutions for the DGLAP equations. Some of them have been used already by the Spin Muon Collaboration (SMC) and the E154 Collaboration, in the most recent analyses of experimental data ([2] and [6], respectively). Approaches [13]–[18] lead to similar results for $g_1(x, Q^2)$[2], which are in contrast with the calculations based on equation (2).

In this article we suggest another method for studying the Q^2 dependence of A_1, based on the observation that the splitting functions of the DGLAP equations for, and the shapes of, the SF g_1 and F_3 are similar for a wide range of x. Our approach allows us to get the Q^2 dependence of A_1 in a simple way (10) and leads to results (some of which have recently been presented ([19])) that are similar to the ones based on the DGLAP evolution.

2 The Q^2 dependence of structure functions

Let us consider the Q^2 evolution of the nonsinglet (NS) and singlet (SI) parts of the SF separately.

For the SF F_3, and the NS parts of g_1 and F_1, the corresponding DGLAP equations can be presented as[3]

$$\frac{dg_1^{NS}(x, Q^2)}{dlnQ^2} = -\frac{1}{2}\gamma^-_{NS}(x, \alpha) \otimes g_1^{NS}(x, Q^2),$$

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[1] Except the leading order of the quark-quark interaction.

[2] The form of the Q^2 dependence for A_1 is different in approaches [13]–[18]. However, all of them are in agreement regarding the weak Q^2 dependence for moderate values of x and the quite strong Q^2 dependence for small values of x.

[3] We use $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$.

$$\frac{dF_1^{NS}(x,Q^2)}{dlnQ^2} = -\frac{1}{2}\gamma^+_{NS}(x,\alpha) \otimes F_1^{NS}(x,Q^2), and \quad (3)$$
$$\frac{dF_3(x,Q^2)}{dlnQ^2} = -\frac{1}{2}\gamma^-_{NS}(x,\alpha) \otimes F_3(x,Q^2),$$

where the symbol $\otimes$ means the Mellin convolution:

$$f_1(x) \otimes f_2(x) \equiv \int_x^1 \frac{dz}{z} f_1(z) f_2(\frac{x}{z}).$$

The splitting functions $\gamma^{\pm}_{NS}(x,\alpha)$ are the reverse Mellin transforms of the anomalous dimensions $\gamma^{\pm}_{NS}(n,\alpha) = \alpha\gamma^{(0)}_{NS}(n) + \alpha^2\gamma^{\pm(1)}_{NS}(n) + O(\alpha^3)$ and the Wilson coefficients[4] $\alpha b^{\pm}(n) + O(\alpha^2)$:

$$\gamma^{\pm}_{NS}(x,\alpha) = \alpha\gamma^{(0)}_{NS}(x) + \alpha^2\left(\gamma^{\pm(1)}_{NS}(x) + 2\beta_0 b^{\pm}(x)\right) + O(\alpha^3), \quad (4)$$

where $\beta(\alpha) = -\alpha^2\beta_0 - \alpha^3\beta_1 + O(\alpha^4)$ is QCD β-function.

Equation (3) shows that the DGLAP equations for F_3 and for the NS part of g_1 are the same (they were obtained exactly in the first two orders of the perturbative QCD[5] [20]) and differ from the one for F_1 already in the first subleading order ($\gamma^{+(1)}_{NS} \neq \gamma^{-(1)}_{NS}$ ([21]), and $b^+_{NS} - b^-_{NS} = (8/3)x(1-x)$).

For the SI parts of g_1 and F_1, the evolution equations are

$$\frac{dg_1^S(x,Q^2)}{dlnQ^2} = -\frac{1}{2}\left[\gamma^*_{SS}(x,\alpha) \otimes g_1^S(x,Q^2) + \gamma^*_{SG}(x,\alpha) \otimes \Delta G(x,Q^2)\right] \quad \text{and}$$
$$\frac{dF_1^S(x,Q^2)}{dlnQ^2} = -\frac{1}{2}\left[\gamma_{SS}(x,\alpha) \otimes F_1^S(x,Q^2) + \gamma_{SG}(x,\alpha) \otimes G(x,Q^2)\right], \quad (5)$$

where the SI splitting functions $\gamma_{Si}(x,\alpha)$, $i = \{S,G\}$ are represented as

$$\gamma_{SS}(x,\alpha) = \alpha\gamma^{(0)}_{SS}(x) + \alpha^2\left(\gamma^{(1)}_{SS}(x) + b_G(x) \otimes \gamma^{(0)}_{GS}(x) + 2\beta_0 b_S(x)\right) + O(\alpha^3) \quad \text{and}$$

$$\gamma_{SG}(x,\alpha) = \frac{e}{f}\left[\alpha\gamma^{(0)}_{SG}(x) + \alpha^2\left(\gamma^{(1)}_{SG}(x) + b_G(x) \otimes \left(\gamma^{(0)}_{GG}(x) - \gamma^{(0)}_{SS}(x)\right) + 2\beta_0 b_G(x) + b_S(x) \otimes \gamma^{(0)}_{SG}(x)\right)\right] + O(\alpha^3), \quad (6)$$

and $e = \sum_i^f e_i^2$ is the sum of the charge squares of f active quarks. Equations for the polarized singlet-splitting functions $\gamma^*_{SS}(x,\alpha)$ and $\gamma^*_{SG}(x,\alpha)$ are similar. They can be obtained via the following replacements (6): $\gamma^{(0)}_{SG}(x) \to \gamma^{*(0)}_{SG}(x)$, $\gamma^{(1)}_{Si}(x) \to \gamma^{*(1)}_{Si}(x)$ and $b_i(x) \to b^*_i(x)$, where $i = \{S,G\}$.

With careful consideration of the quark parts of (5) and (6) we see that the value of $b^*_s(x)$ ($b_s(x)$) is the same as that of $b^-(x)$ ($b^+(x)$). The difference between $\gamma^{-(1)}_{NS}(x)$ and $\gamma^{*(1)}_{SS}(x) + b^*_G(x) \otimes \gamma^{(0)}_{GS}(x)$ is negligible because there is not a power singularity at $x \to 0$ (i.e., a singularity at $n \to 1$ in momentum space). Moreover, this difference decreases at $O(1-x)$ as $x \to 1$ ([24]). (In contrast, the difference between $\gamma^{(1)}_{SS}(x) + b_G(x) \otimes \gamma^{(0)}_{GS}(x)$ and $\gamma^{*(1)}_{SS}(x) + b^*_G(x) \otimes \gamma^{(0)}_{GS}(x)$ contains a power singularity at $x \to 0$ (see for example [20]).) Thus, the DGLAP equations for F_3 and the SI part of g_1 have close splitting functions, which are essentially different from the splitting functions of the SI part of F_1.

The quark part of the SI of SF g_1 itself contains two components, valence and sea. The valence part does not connect with the gluon, and obeys the DGLAP equation similar to the first equation in (3). The sea part obeys the first equation in (5), but its value seems to be quite small, since it has not yet been observed experimentally.

The gluon distribution in g_1 is not so important for the modern data, ([14,15,9,18,22]), in contrast with the unpolarized case; data are described well for extremely different values of $\Delta G(x,Q^2)$ and even for different signs of its first moment. Hence we will neglect this term in our analysis.

Thus, the valence component seems to dominate in the SI part of g_1 for the range of the present experimental data[6], and it allows us to expect a similarity between the SI part of g_1 and the SF F_3.

As we saw above, the shapes and DGLAP equations for g_1 and F_3 are very similar in the NS and SI analyses[7], and both of them differ from the corresponding equations for F_1. This similarity suggests that the Q^2 dependences for SF $g_1(x,Q^2)$ and $F_3(x,Q^2)$ are also similar.

[4] We consider here structure functions but not parton distributions. Note also that $b^+_{NS}(n)$ and $b^-_{NS}(n)$ can be defined as $b_{1,NS}(n) = b_{2,NS}(n) - b_{L,NS}(n)$ and $b_{3,NS}(n)$, respectively.

[5] This is easy to demonstrate in any order of the perturbation theory. The SFs g_1 and F_3 are the results of the γ_5 matrix contribution to the lepton and hadron parts of deep inelastic cross-sections, respectively. In the NS case there is only one γ-matrix trace, connecting the lepton and hadron parts. Its contribution $\sim\ tr(\gamma_5\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta....)$ is the same in both cases above. For the SI part of g_1, there are diagrams with several traces, which arise in the second order of perturbation QCD and lead to the difference between the splitting functions of SF F_3 and the SI part of SF g_1.

[6] For support of this point of view, see also the recent analysis by the E154 Collaboration, where the contributions of the sea + gluon parts and the valence parts are divided, studied, and presented in Fig. 2 ([6]).

[7] The similar shapes of SF F_3 and g_1 in the range of measured values of x can be seen also in [25].

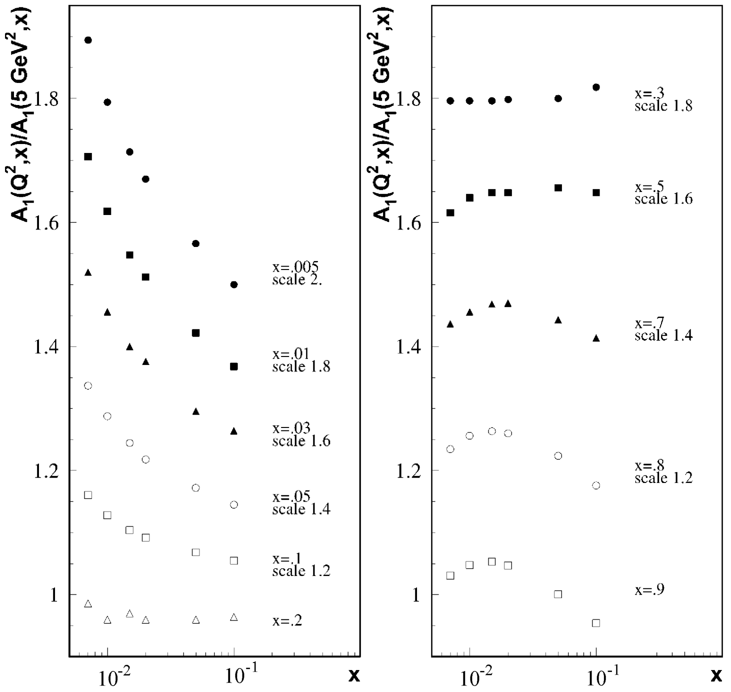

Fig. 1. Q^2 dependence of the ratio $A_1(x,Q^2)/A_1(x,5GeV^2)$

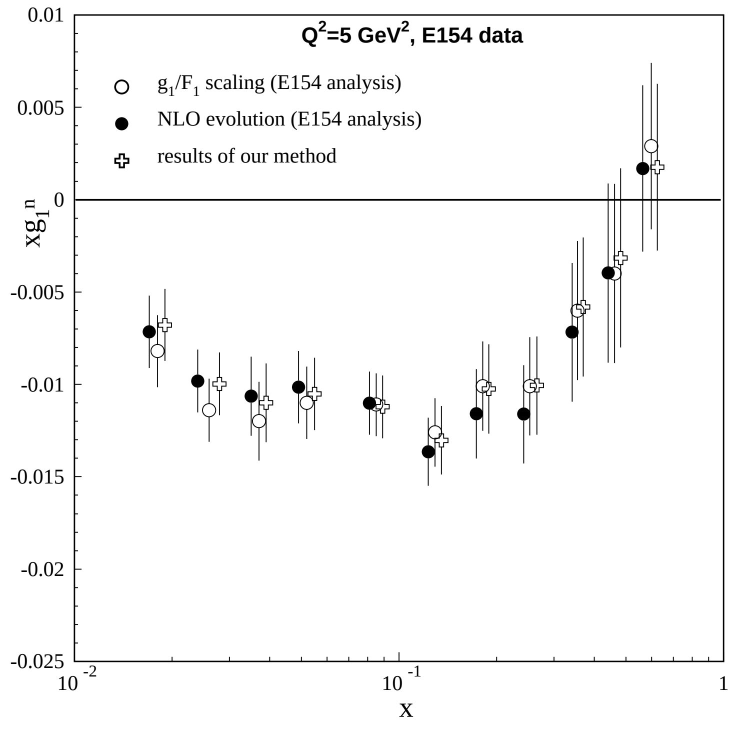

Fig. 2. Structure function $xg_1^n(x,Q^2)$, evolved to $Q^2 = 5GeV^2$ using equation (10); DGLAP NLO evolution; the assumption that g_1^n/F_1^n is independent of Q^2. The last two sets are taken from [6]

The similarity between the Q^2 dependence of $F_3(x,Q^2)$ and that of $g_1(x,Q^2)$ may be supported also by some arguments following from an analysis at $x \to 0$[8]. Although all existing data in polarized DIS (excluding the first two SMC points) are outside the region $x \leq 10^{-2}$, the study of small x asymptotics is important for future data, as well as for an extrapolation of the present data from the small x region.

The similarity of the splitting functions of SF F_3 and g_1 have already been demonstrated; thus, we now turn to the shapes of F_3 and g_1 at small x. It is well known that SF F_3 is governed, for small values of x, by the ρ-meson trajectory; thus,

$$F_3(x) \sim x^{-1/2}. \qquad (7)$$

[8] At $x \to 1$ the behaviours of g_1 and F_3 (and F_1, too) should be similar because they are governed by valence quark distributions.

The Q^2 evolution does not change this behaviour. For SF g_1, the situation is not so clear. Regge-like analysis [26] shows that the NS part of g_1 is governed by an a_1 trajectory, i.e., $g_1(x) \sim x^{-\alpha_P(a_1)}$, with the intercept $\alpha_P(a_1)$ having values $0 \geq \alpha_P(a_1) \geq -1/2$ ([27]). However, the BFKL-inspired approach ([28]) leads to more singular behaviour for the NS part,

$$g_1^{NS}(x) \sim x^{-0.45}, \quad (8)$$

which is close to (7). For the SI part of g_1, the information given is very poor. The BFKL-inspired approach ([29]) concludes that for small x, $g_1^{SI}(x) \sim x^{-1}$, yet when in reality the SI part of g_1[9] has not been observed at small x. Indeed, the deutron SF $g_1^d(x)$, which is close to the SI component, is comparable with zero at small x.

As a consequence, the shapes of the SF F_3 and the NS part of SF g_1 seem to be close for small values of x also (if the BFKL approach is correct). The SI part of g_1 may have another shape, but modern experimental data do not allow for its study.

The analysis discussed above allows us to conclude that the function A_1^*, defined as

$$A_1^*(x) = \frac{g_1(x,Q^2)}{F_3(x,Q^2)}, \quad (9)$$

has to be practically Q^2-independent in the whole region of modern experimental data ([1]-[6]).

Consistent with (9), the measured asymmetry $A_1(x_i, Q_i^2)$ can be found for some value of Q^2, as:

$$A_1(x_i,Q^2) = \frac{F_3(x_i,Q^2)}{F_3(x_i,Q_i^2)} \cdot \frac{F_1(x_i,Q_i^2)}{F_1(x_i,Q^2)} \cdot A_1(x_i,Q_i^2). \quad (10)$$

3 Calculation of $A_1(x,Q^2)$ and $\Gamma_1(Q^2)$

To apply this approach, we used the SMC ([2]), E143 ([4]), and E154 ([5,6]) Collaboration data[10]. To use the relation in (10) we parametrized the CCFR data on $F_2(x,Q^2)$ and $xF_3(x,Q^2)$ [30] in the same form as the NMC fit of the structure function $F_2(x,Q^2)$ ([31]) (see Appendix). To obtain SF $F_1(x,Q^2)$, we take the parametrization of the CCFR data on $F_2(x,Q^2)$ ([30]) and the SLAC parametrization of $R(x,Q^2)$ ([32]) and use the relation

$$F_1(x,Q^2) = \frac{F_2(x,Q^2)}{2x(1+R(x,Q^2))} \cdot (1 + \frac{4M^2x^2}{Q^2}), \quad (11)$$

where M is the proton mass.

Using parametrizations of CCFR data ([30]) for both SF $xF_3(x,Q^2)$ and $F_2(x,Q^2)$ in (10) allows us to avoid systematical uncertainties and nucleon correlation in nuclei. Figure 1 shows the ratio $A_1(Q^2)/A_1(5GeV^2)$ obtained in (10). Comparison of Fig. 1 with the results of the E154 Collaboration (Fig. 4 in [6]) shows reasonable agreement.

[9] It is the sea component of the SI part of g_1, that dominates here if it has nonzero magnitude.

[10] A similar analysis of the SMC data in [1] has been done in [33] using old CCFR data ([35]).

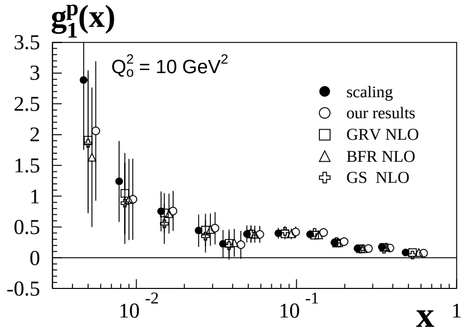

Fig. 3. Structure function $xg_1^p(x,Q^2)$, evolved to $Q^2 = 10GeV^2$ using equation (10); the assumption that g_1^p/F_1^p is independent of Q^2; DGLAP NLO evolution according to the analyses of [13,14]. The last two sets are taken from [2]

The SF $g_1(x,Q^2)$ was evaluated using (1), in which the spin-averaged SF F_1 has been calculated using the NMC parametrization of $F_2(x,Q^2)$ ([31]). Results are presented in Fig. 2 and Fig. 3 for the E154 and SMC data, respectively. Our results are in excellent agreement with the SMC and E154 Collaboration analyses that are based on direct DGLAP evolution (see [2] and [6], respectively).

For comparison with the theory predictions on the sum rules, we have calculated the first moment value of the structure function g_1 for different Q^2 values,

$$\Gamma_1 = \tilde{\Gamma}_1 + \Delta\tilde{\Gamma}_1, \quad (12)$$

where

$$\tilde{\Gamma}_1(Q^2) = \int_{x_{min}}^{x_{max}} g_1(x,Q^2)dx \quad \text{and} \quad (13)$$

$$\Delta\tilde{\Gamma}_1(Q^2) = \int_0^{x_{min}} g_1(x,Q^2)dx + \int_{x_{max}}^1 g_1(x,Q^2)dx$$

are the integrals of the measured kinematical x region, and an estimation for unmeasured ranges, respectively.

The value of $\Delta\tilde{\Gamma}_1$ from the unmeasured x-regions was estimated using original methods by owner collaborations. We note here that the method of $\Delta\tilde{\Gamma}_1$ estimation may lead to some underestimation of $g_1^{p,d,n}(x,Q^2)$ for small values of x and, consequently, of $\Gamma^{p,d,n}(Q^2)$ (see the careful analysis in [14]). To clear up this problem it is necessary to have more precise data for small values of x.

Values of $\Gamma_1(Q^2)$, which are obtained from the exact solution of the DGLAP evolution equation ([2,6]) of $g_1(x,Q^2)$ and in our approach on the scaling of A_1^*, are quite close to each other for all cases discussed here. Thus all approaches lead to similar conclusions for $\Gamma^{p,d,n}(Q^2)$ and the results, in turn, strongly disagree with the theoretical predictions in [36]. Hence we will consider the effect of A_1^* scaling only for the Bjorken Sum Rule $\Gamma_1^p - \Gamma_1^n$.

SMC and E143 deutron data give us the value of Γ_1^n,

$$\Gamma_1^p + \Gamma_1^n = \frac{2\Gamma_1^d}{1 - 1.5w_d}, \quad (14)$$

Table 1. The values of $\Gamma_1^p - \Gamma_1^n$. Errors are shown only for certain points in each set of data. Uncertainties in our analysis are comparable with ones in [2–4,6]

Q^2 (GeV^2)	100	30	10	5	3
SMC proton [2] and deutron data [3]					
A_1-scaling	0.247	0.226	0.202	0.186	0.170
A_1^*-scaling	0.210	0.201	0.191	0.184	0.176
Analysis of [2]			0.183	0.181 ± 0.035	
SMC proton [2] and E154 neutron data [6]					
A_1-scaling	0.221	0.209	0.194	0.183	0.171
A_1^*-scaling	0.194	0.190	0.185	0.181	0.176
E143 proton and deutron data [4]					
A_1-scaling	0.170	0.169	0.165	0.160	0.154
A_1^*-scaling	0.163	0.162	0.160	0.157	0.154
Analysis of [4]				0.164 ± 0.021	0.164
E143 proton ([4]) and E154 neutron data ([5,6])					
A_1-scaling	0.189	0.186	0.179	0.174	0.169
A_1^*-scaling	0.172	0.172	0.171	0.169	0.166
Analysis of [6]				0.171 ± 0.013	
Analysis of [4]				0.170 ± 0.012	
Theory	0.194	0.191	0.186	0.181 ± 0.002	0.177

where w_d=0.05 is the probability of the deutron being in a D-state. Knowledge of proton and neutron first momenta $\Gamma_1^{p,n}$ allows us to test the Bjorken Sum Rule:

$$\Gamma_1^{p-n} \equiv \int_0^1 (g_1^p(x,Q^2) - g_1^n(x,Q^2))dx = \Gamma_1^p - \Gamma_1^n \qquad (15)$$

Table 1 compares these results with values published by the SMC, E143 and E154 Collaborations and with the theoretical predictions computed in the third order of the QCD α_s ([37]).

Let us now describe the main results, which follow from Table 1 and Figs. 1–3.

- Our description of the Q^2 evolution of the asymmetry $A_1(x,Q^2)$ has a very simple form (10) but its results are consistent with powerful analyses ([14,15,6]).
- Results concerning $g_1(x,Q^2)$ are in excellent agreement with the SMC and E154 Collaborations' analyses, based on direct DGLAP evolution.
- Our method allows us to test sum rules simply and accurately. Obtained results on the $\Gamma_1^p - \Gamma_1^n$ show that experimental data strongly confirm the Bjorken Sum Rule prediction.

4 Conclusion

We have considered the Q^2 evolution of the asymmetry $A_1(x,Q^2)$ based on the similarity between the Q^2 dependences of the SFs $g_1(x,Q^2)$ and $F_3(x,Q^2)$[11]. Obtained results on $g_1(x,Q^2)$ are in very good agreement with the corresponding results of the SMC and E154 Collaborations, based on direct DGLAP evolution. Our tests of the Ellis–Jaffe Sum Rules for the proton, deutron and neutron yield results that are very close to the values published by the Spin Muon, E143 and E154 Collaborations. However, the variations of sum rule values coming due to the Q^2 evolution of asymmetry $A_1(x,Q^2)$ have opposite signs for the proton and deutron. This increases the similarity between the experimental results and the theoretical predictions regarding the Bjorken Sum.

We believe that future, more precise data will illuminate a violation of our hypothesis (probably for very small x values: $x \leq 10^{-3}$). This violation will indicate clearly the appearance of nonzero contributions from the sea quark and gluon components of SF $g_1(x,Q^2)$, which should have quite singular shapes at small values of x (see the careful analysis in [38]).[12] Thus, checking the Q^2 dependence of the ratio $A_1^* = g_1/F_3$ against future precise data will allow for a more a qualitative estimation of the shapes and the Q^2 dependences of gluon and sea quark distributions.

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Appendix

The parametrizations used for CCFR data [30] are:

$$xF_3(x,Q^2) = F_3^a \cdot \left(\frac{log(Q^2/\Lambda^2)}{log(Q_0^2/\Lambda^2)}\right)^{F_3^b} \quad \text{and}$$

$$F_2(x,Q^2) = F_2^a \cdot \left(\frac{log(Q^2/\Lambda^2)}{log(Q_0^2/\Lambda^2)}\right)^{F_2^b},$$

where

$$F_3^a = x^{C_1} \cdot (1-x)^{C_2} \cdot \Big(C_3 + C_4 \cdot (1-x) + C_5 \cdot (1-x)^2 + C_6 \cdot (1-x)^3 + C_7 \cdot (1-x)^4\Big),$$

$$F_2^a = x^{B_1} \cdot (1-x)^{B_2} \cdot \Big(B_3 + B_4 \cdot (1-x) + B_5 \cdot (1-x)^2 + B_6 \cdot (1-x)^3 + B_7 \cdot (1-x)^4\Big),$$

[11] The useful parametrizations of SF $F_2(x,Q^2)$ and $xF_3(x,Q^2)$ are obtained for new CCFR data and presented in the Appendix.

[12] The separation and study of SI and NS components with similar shapes will require an elaborate procedure.

Table 2. The values of the coefficients of CCFR data parametrization

C_1	C_2	C_3	C_4	C_5	C_6
0.33092	3.5000	6.5739	-7.1015	2.5388	7.6944
C_7	C_8	C_9	C_{10}	C_{11}	
-8.4285	4.9135	-4.9857	-8.1629	1.8193	
B_1	B_2	B_3	B_4	B_5	B_6
-0.06101	3.5000	4.9728	-3.1309	-1.3361	0.94242
B_7	B_8	B_9	B_{10}	B_{11}	
0.11729	-0.92024	-1.6489	0.61776	0.38910	

$$F_3^b = C_8 + C_9 \cdot x + \frac{C_{10}}{x + C_{11}},$$
$$F_2^b = B_8 + B_9 \cdot x + \frac{B_{10}}{x + B_{11}},$$

and $Q_0^2 = 20$ GeV^2, $\Lambda = 337$ MeV. The values of Q_0^2 and Λ are fixed, consistent with the CCFR analysis ([30]). The values of the coefficients C_i $(i = 1, ..., 11)$ and B_i $(i = 1, ..., 15)$ are given in Table 2.

References

1. SMC, D. Adams, et al., Phys. Lett. B **329**, 399 (1994), B **339**, 332(E) (1994), Phys. Lett. B **357**, 248 (1995); E143 Collab., K. Abe, et al., Phys. Rev. Lett. **74**, 346 (1995), Phys. Rev. Lett. **75**, 25 (1995); HERMES Collab., K. Ackerstaff, et al., Phys. Lett. B **404**, 383 (1997)
2. SMC, D. Adams, et al., Phys. Rev. D **56**, 5330 (1997)
3. SMC, D. Adams, et al., Phys. Lett. B **396**, 338 (1997)
4. E143 Collab., K. Abe, et al., Phys. Rev. D **58**, 112003 (1998)
5. E154 Collab., K. Abe, et al., Phys. Rev. Lett. **79**, 26 (1997)
6. E154 Collab., K. Abe, et al., Phys. Lett. B **405**, 180 (1997)
7. N. Gagunashvili, et al., NIM **412**, 146 (1998)
8. M. Anselmino, A. Efremov, E. Leader, Phys. Rep. **261** (1995) 1; J. Ellis, M. Karliner, CERN-TH.279/95 and TAUP-2297-95, 1995 (unpublished); B.L. Ioffe, in Proceedings of Quarks-94, Vladimir, p.14
9. G.P. Ramsey, Prog. Part. Nucl. Phys. **39**, 599 (1997)
10. J.D. Bjorken, Phys. Rev. **148**, 1467 (1966); D **1**, 1376 (1970)
11. J. Ellis, R.L. Jaffe, Phys. Rev. D **9**, 1444 (1974); D **10**, 1669 (1974)
12. J. Ellis, M. Karliner, Phys. Lett. B **313**, 131 (1993); F.E. Close, R.G. Roberts, Phys. Lett. B **316**, 165 (1993)
13. G. Altarelli, P. Nason, G. Ridolfi, Phys. Lett. B **320**, 152 (1994); M. Gluck, E. Reya, W. Vogelsang, Phys. Lett. B **359**, 201 (1995); T. Gehrmann, W.J. Stirling, Z. Phys. C **65**, 470 (1995); E143 Collab., K. Abe, et al., Phys. Lett. B **364**, 61 (1995); R.D. Ball, S. Forte, G. Ridolfi, Nucl. Phys. B **444**, 287 (1995); A.V. Kotikov, D.V. Peshekhonov, Phys. Rev. D **54**, 3162 (1996), Phys. of Atomic Nuclei, **60**, 653 (1997); T. Weigl, W. Melnitchouk, Nucl. Phys. B **465**, 267 (1996); G. Altarelli, et al., Nucl. Phys. B **496**, 337 (1997); J.P. Nassalski, in Proceedings of the XXVII International Conference on High Energy Physics (1996) p. 35 (hep-ph/9612352)
14. M. Gluck, E. Reya, M. Stratmann, W. Vogelsang, Phys. Rev. D **53**, 4775 (1996)
15. T. Gehrmann, W.J. Stirling, Phys. Rev. D **53**, 6100 (1996)
16. R.D. Ball, S. Forte, G. Ridolfi, Phys. Lett. B **378**, 225 (1996)
17. M. Hirai, S. Kumano, M. Miyama, Comp. Phys. Commun. **108**, 38 (1998)
18. W.-D. Nowak, A.V. Sidorov, M.V. Tokarev, Nuovo Cim. A **110**, 757 (1997); E. Leader, A.V. Sidorov, D.B. Stamenov, Int. J. Mod. Phys. A **13**, 5573 (1998)
19. A.V. Kotikov, D.V. Peshekhonov, in Proceedings of the DIS98 Workshop, p. 612 (hep-ph/9805374)
20. J. Kodaira, et al., Phys. Rev. D **20**, 627 (1979); Nucl. Phys. B **159**, 99 (1979)
21. D.A. Ross, C.T. Sachrajda, Nucl. Phys. B **149**, 497 (1979); G. Altarelli, Phys. Rep. **81**, 1 (1982)
22. M. Goshtasbpour, G.P. Ramsey, Phys. Rev. D **55**, 1244 (1997); L. E. Gordon, M. Goshtasbpour, G.P. Ramsey, Phys. Rev. D **58**, 094017 (1998)
23. FNAL E581/704 Collab., D.L. Adams, et al., Phys. Lett. B **336**, 369 (1994)
24. R. Merting, W.L. van Neerven, Z. Phys. C **70**, 625 (1996); W. Vogelsang, Phys. Rev. D **54**, 2023 (1996)
25. C. Bourrely, J. Soffer, Nucl. Phys. B **445**, 341 (1995)
26. R.L. Heinmann, Nucl. Phys. B **64**, 429 (1973)
27. J. Ellis, M. Karliner, Phys. Lett. B **213**, 73 (1988)
28. J. Bartels, B.I. Ermolaev, M.G. Ryskin, Z. Phys. C **70**, 273 (1996)
29. J. Bartels, B.I. Ermolaev, M.G. Ryskin, Z. Phys. C **72**, 627 (1996)
30. CCFR/NuTeV Collab., W. Seligman, et al., Phys. Rev. Lett. **79**, 1213 (1997); W. Seligman, Ph.D. thesis, Columbia University, Nevis Report 292, 1997
31. NMC, M. Arneodo, et al., Phys. Lett. B **364**, 107 (1995)
32. L.W. Whitlow, et al., Phys. Lett. B **250**, 193 (1990)
33. A.V. Kotikov, D.V. Peshekhonov, JETP Lett. **65**, 7 (1997); in Proceedings of the DIS96 Workshop, p.612 (hep-ph/9608369)
34. P. Ratchliffe, in Proceedings of the International Symposium SPIN96, p.161 (hep-ph/9611348)
35. CCFR Collab., P.Z. Quintas, et al., Phys. Rev. Lett. **71**, 1307 (1993); M. Shaevitz, et al., Nucl. Phys. Proc. Suppl. B **38**, 188 (1995)
36. S.A. Larin, T. van Ritbergen, J.A.M. Vermaseren, Phys. Lett. B **404**, 153 (1997)
37. S.A. Larin, J.A.M. Vermaseren, Phys. Lett. B **259**, 345 (1991)
38. D. van Harrach, W.-D. Nowak, J. Soffer, Preprint DESY 97-232, CPT-97/P.3564, 1997 (hep-ph/9712207); A. De Roeck, T. Gehrmann, DESY 97-233, 1997 (hep-ph/9711512)